

# massive star formation

## Massive stars

What is a massive star? → see table.

**Importance:** They are the principal source of heavy elements and UV radiation. They provide a source of mixing and turbulence in the interstellar medium [ISM] (winds, outflows, expanding HII regions, supernova explosions).

**Galactic magnetic fields:** plasma ejections like strong stellar winds and supernova explosions produce seed magnetic fields for the galactic dynamo (Fleishman & Toptygin 2013, § 8.7.1). The galactic dynamo is turbulent and fed by differential rotation ( $\alpha\Omega$ -dynamo). The observed galactic magnetic field is weak ( $\sim 10^{-5} B_{\oplus}$ ), but of dynamical importance. Apart from a dynamo, another possible origin of the magnetic field are cosmic rays.

Term	Mass	Observations
Massive star	$> 8M_{\odot}$	OB star, massive enough to produce a type II supernova. (O, B0, B1)
Very massive star	$[10^2, 10^3]M_{\odot}$	Unlikely to be formed in the present epoch.
Ultramassive star	$[10^3, 10^4]M_{\odot}$	
Supermassive star	$[10^4, 10^8]M_{\odot}$	equilibrium dominated by radiation pressure. Collapse due to GR instability. Also unlikely to be formed in the present epoch.

**Heating and cooling the ISM:** Cosmic rays, UV radiation and dissipation of turbulence are the sources of heating in the ISM, and heavy elements found in dust and molecules are responsible for cooling, promoting the generation of later generations of stars (although ionization and heating from the massive star formation [MSF] process can suppress subsequent star formation in the neighborhood).

∴ Massive stars profoundly affect (in different scales) the star and planet formation process.

**Angular momentum problem:** when a cloud contracts, the conservation of angular momentum would imply that any small rotation in the cloud will get amplified as it contracts. However, this is not what is observed, which means that angular momentum is lost somehow. Two mechanisms that can explain this angular momentum loss are a) turbulent velocity profile and b) magnetic braking (if there is a core with external magnetic field, magnetic tension can exert torque and transfer angular momentum to the surroundings).

## Additional notes on star formation

**Protostellar initial conditions for low-mass stars:** composition: 70% of mass is hydrogen in molecular form, so  $\bar{m} \approx 2.37 u$ , core mass: few solar masses, and radius of 0.05–0.1 pc. Initial density distribution: a) uniform, b) power law  $\rho \propto r^{-n}$  (n: observational), c) Bonnor-Ebert sphere (flat at the center, approaching  $\rho \propto r^{-2}$  in the outer regions. The core is nearly isothermal at  $T \sim 10$  K, and the ratio of thermal to gravitational energy is  $\alpha \approx 0.4$ .

**Bonnor-Ebert sphere:**

Speed of sound in an ideal gas:  $\{PV = Nk_B T, M = \bar{m} N\} \implies P = (k_B T / \bar{m}) \rho$ , but  $c_s = \sqrt{P/\rho} \implies c_s^2 = \sqrt{k_B T / \bar{m}}$  or, if  $\mu = \bar{m} N_A$  is the mean molecular mass in u,  $c_s^2 = \sqrt{R_g T / \mu}$ , where  $R_g$  is the ideal gas constant. This also means that the equation of state is  $P = c_s^2 \rho$ .

Structure equations, from [Notes on Astrophysics § Classical structure of compact objects], except that the equation of state is the ideal gas instead of a polytrope:

$$(a) \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2}{\rho} \frac{dP}{dr} \right) = -4\pi G \rho \implies \frac{1}{r^2} \frac{d}{dr} \left( r^2 c_s^2 \frac{d \ln \rho}{dr} \right) = -4\pi G \rho,$$

$$(b) P = c_s^2 \rho.$$

Similarly as we did in "Notes on Astrophysics", we change variables to  $\xi = \frac{\sqrt{4\pi G \rho_c}}{c_s} r := r/a$  and  $-\theta = \ln(\rho/\rho_c)$ ,

so we get the equation  $\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) = e^{-\theta}$ , with the boundary conditions  $\theta(0) = 0$  and  $(d\theta/d\xi)(0) = 0$ .

**Solution:** as we did before with the Lane-Emden equation,

$M = \int_0^R 4\pi r^2 \rho dr = 4\pi a^3 \rho_c \int_0^{\xi_1} \xi^2 e^{-\theta} d\xi$ , where  $\xi_1$  is the adimensional radius of the star, considered as when the adimensional pressure goes to zero. Also, as before, with the adimensional equation, we substitute  $e^{-\theta}$  in the mass and

get  $M = 4\pi a^3 \rho_c \int_0^{\xi_1} d\xi \xi^2 \left[ \frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\theta}{d\xi} \right) \right] = 4\pi a^3 \rho_c \xi_1^2 \theta'(\xi_1)$ . The

solution as function of  $\xi_1$  are shown in the plot. A program analog to the one presented in "Notes on Astrophysics" was used (modifications: initial conditions:  $\theta(0) = 0$ ,  $(d\theta/d\xi)(0) = 0$ ; stopping condition: while  $\xi_1 < 12$ , results are printed in each step).

*Values at the surface:* From the mass at  $\xi_1$  we can solve for  $\rho_c$ :

$$\rho_c^{1/2} = \frac{4\pi \xi_1^2 \theta'(\xi_1)}{M(\xi_1)} \frac{c_s^3}{(4\pi G)^{3/2}}. \text{ With that, we can calculate the radius}$$

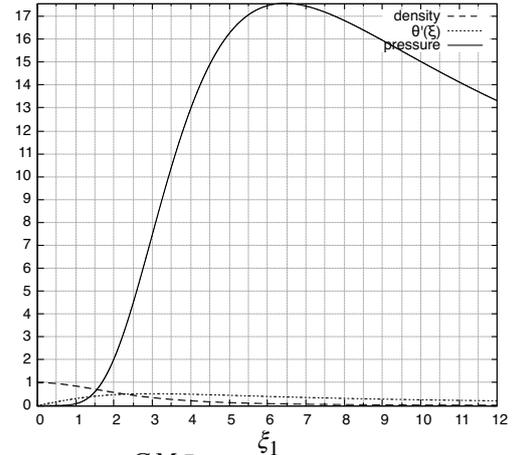
$$R = a\xi = \frac{M(\xi_1)G}{c_s^2 \xi_1^2 \theta'(\xi_1)}, \text{ the density } \rho(\xi_1) = \rho_c e^{-\theta} = \frac{\xi_1^4 [\theta'(\xi_1)]^2 e^{-\theta_1}}{4\pi G^3 M(\xi_1)} c_s^6,$$

$$\text{and the pressure } P_1 = \rho_1 c_s^2.$$

*Critical values:* from the plot, the external pressure reaches a maximum at

$$\xi_{1,\text{crit}} \approx 6.5, \text{ and the equilibrium is marginally stable. At that point, } R_{\text{crit}} \approx 0.41 \frac{GM\bar{m}}{k_B T} \text{ and}$$

$P \approx 1.40 \frac{c_s^8}{G^3 M^2}$ . This is consistent with the Jeans values. The limit  $\xi_1 \rightarrow \infty$  is the *singular isothermal sphere*, with  $\rho \rightarrow r^{-2}$ ; the outer radius is infinite, infinite central density, unstable equilibrium.



**Infall rate:**  $\dot{M} \sim M_{\text{core}}/t_{\text{ff}}$ . In [Notes on Astrophysics, § Star formation], we saw that  $t_{\text{ff}} \sim (G\rho)^{-1/2}$ . Introducing  $R_{\text{crit}}$  from the Bonnor-Ebert sphere and  $c_s^2 = k_B T/\bar{m}$  in  $\rho \sim M_{\text{core}}/R_{\text{core}}^3$ , we get  $\rho \approx c_s^6/(M_{\text{core}}^2 \cdot 0.4^3 \cdot G^3)$ . Now,  $t_{\text{ff}} \sim GM_{\text{core}}/c_s^3$  and  $\dot{M} \sim c_s^3/G$ .

**The virial parameter:** (extension of Notes on Astrophysics § Star formation ¶ Virial temperature) The virial theorem implies

$$1 = -\frac{2\langle K \rangle}{\langle U \rangle} = \frac{-2 \cdot 5 \cdot \frac{1}{2} M \langle v^2 \rangle R}{3GM^2 \bar{m}}, \text{ but } 3\langle v_x^2 \rangle = \langle v^2 \rangle \text{ and then we can define } \alpha_{\text{vir}} = \frac{5\langle v_x^2 \rangle R}{GM}.$$

**First Larson core:** in a collapsing core, the point in which the dust becomes opaque to its own radiation (radiation cannot escape).

**Thermal re-emission, diffusive regime:** for a radiative star (Notes on Astrophysics § Stellar structure ¶ Energy transport in radiative zones), the diffusion approximation can be used. An emitted photon is absorbed, re-emitted and scattered, a large amount of times before being transmitted through the material. Then, as the photons diffuse to lower temperatures, they are re-emitted like a blackbody at the local temperature.

**Thermal adjustment time:** the timescale for significant changes in the thermal profile of a radiative star is the thermal adjustment time, and can be approximated by the Kelvin-Helmholtz timescale (radiation of gravitational energy).

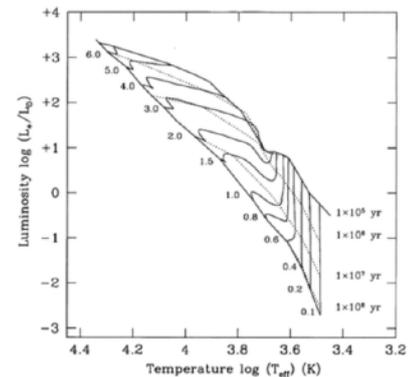
**Pre-main-sequence evolutionary tracks:**

*Initial parameters:* mass, chemical composition,  $l/H$  (=mean free path of largest convective elements/pressure scale height), initial model.

*Hayashi track* ( $<0.5 M_{\odot}$ ): Convective phase. The evolution in the HR diagram is vertical and downwards. Energy transport is quite efficient in the interior; the rate of energy loss is determined by a thin radiative layer at the surface.

*Heney track* ( $>0.5 M_{\odot}$ ): Radiative phase. The evolution in the HR diagram is relatively horizontal. Contraction  $\Rightarrow$  T interior increases  $\Rightarrow$  opacity decreases, the interior becomes radiative (convection stops). Luminosity is no longer controlled by the surface layer, but by the opacity of the whole radiative region. Sharp bend to the left in the evoluc. track. During the track: short contraction times, approximately the Kelvin-Helmholtz timescale (relatively constant luminosity).

*Transition to the main sequence:* When H starts burning (main sequence), the contraction stops (and for high masses, the luminosity increases gradually, for a solar mass track, the luminosity decreases slightly).



# Massive star formation

**Differences between massive and low-mass star formation:** The accretion time of the envelope for massive stars is longer than the gravitational contraction time, that is, the core can reach the main sequence while accretion is still going on. MRI doesn't apply, and the main mechanisms for angular momentum transport is gravity.

**Massive star present populations:** massive stars form primarily in clusters, along with numerous low-mass stars. Four categories: 1) gravitationally bound OB clusters (eg., Orion Nebula cluster), 2) OB associations ([110] pc apart, not gravitationally bound), 3) "runaway" OB stars (individual stars with velocities of 40 km/s or more, probably ejected from a cluster) 4) untraceable to a cluster (<10% of all massive stars).

**Initial conditions (typical numbers) for massive stars:** mass of the core:  $100M_{\odot}$ , mean density:  $4 \cdot 10^{-15} \text{ kg m}^{-3}$ , temperature: 15 K. For these numbers, the thermal Jeans mass is less than  $1M_{\odot}$ . The virial parameter is of order unity at the beginning.

**Accretion rate (order of magnitude):** consider a cloud of  $M \sim 200M_{\odot}$ ,  $R \sim 0.1 \text{ pc}$ . Its free-fall time is  $t_{ff} \sim (G\rho)^{-1/2} \sim (GM/R^3)^{-1/2}$ . The accretion rate is  $\dot{M} \sim M/t_{ff}$ . Inserting the numbers, we get around  $10^{-3} M_{\odot}/\text{yr}$ .

**Dependence of the accretion rate with time:** consider a cloud of density profile  $\rho \propto r^{\beta}$ . The enclosed mass in a sphere of radius  $r$  is  $M_r \propto \rho V \propto r^{\beta+3}$ . the freefall time is  $t_{ff} \sim (G\rho)^{-1/2} \propto r^{-\beta/2}$ . The accretion rate is  $\dot{M} = M/t_{ff} \propto r^{\beta+3+\beta/2}$ . Notice that for  $\beta = -2$ ,  $\dot{M} = \text{const}$ , which means that as the cloud contracts, the accretion rate does not change. Other slopes for the density profile lead to a variable accretion rate over time.

**Core fragmentation:** as the collapse proceeds, thermal effects dominate, the Jeans mass is lowered and the core fragments in several pieces.

**Formation of the stellar core:** after the isothermal collapse phase (central density increases for around 6 orders of magnitude), the adiabatic collapse phase enters and heats up the center up to 2000 K, when the hydrogen dissociation kicks in and a second collapse begins. After that, the central temperature reaches 20 000 K and the stellar core begins to form in hydrostatic equilibrium. The accretion phase begins.

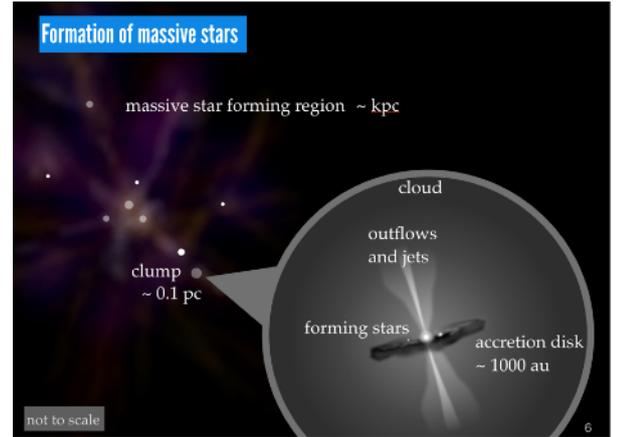
## The problem of radiation pressure:

When the main sequence is reached, radiation from the core surface shifts to the UV region, producing ionizing photons. These photons may ionize the dusty infalling gas, and this effect must be taken into account since it slows down infall. Opacity of the dust in the infalling envelope: {UV, optical: high; IR: lower}, however, dust species evaporate above 1500 K (dust destruction front; eg.  $\sim 8 \text{ AU}$  for  $L \sim 1000L_{\odot}$ , scales as  $L^{1/2}$ ).

**Opacity:** We define the mean free path of light in a given frequency as  $\ell_{\nu} = 1/(\kappa_{\nu}\rho)$ , where  $\kappa_{\nu}$  is called *opacity*  $\mathcal{D}[\kappa] = L^2 M^{-1}$ . This is analogy to particle collisions, where  $\ell = 1/(\sigma n)$ , where  $\sigma$  is the scattering cross section and  $n$  the number density of particles. Also, from [Notes on Astrophysics § Radiative transfer ¶ Radiation pressure], we know that  $dE_{\nu} = F_{\nu} dA dt$ , which implies that the momentum is  $dE_{\nu}/c = F_{\nu} dA dt/c$ . Pressure = momentum/( $dA dt$ ) =  $F_{\nu}/c$ .

**Gravity against radiation pressure:**  $\frac{GMdm}{r^2} = dPdA \implies \frac{GM}{r^2} = \frac{dPdA}{\rho dr dA}$ . Taking  $dr$  as the free path of a light ray, we have  $\frac{GM}{r^2} = \frac{\bar{\kappa} F}{c}$ , where  $\bar{\kappa} = \frac{1}{F} \int_0^{\infty} \kappa_{\nu} F_{\nu} d\nu$

**Mass limit for a star in a dust envelope:** since  $L_{\star} = 4\pi r^2 F$ , we have  $\frac{L_{\star}}{M_{\star}} = \frac{4\pi Gc}{\bar{\kappa}}$ . For dust at  $T \sim [300, 1000] \text{ K}$ ,  $\bar{\kappa} \approx 0.8 \text{ m}^2/\text{kg}$ , which gives us  $L_{\star}/M_{\star} = 0.31429$ , or expressed in solar units,  $(L_{\star}/L_{\odot})/(M_{\star}/M_{\odot}) \approx 1600L_{\odot}/M_{\odot}$  which corresponds to a main-sequence star of about  $15M_{\odot}$ . According to this calculation, no massive stars should be formed! Disregarding dust, the mass limit, which becomes the Eddington limit, becomes  $\sim 200M_{\odot}$ . If the star is accreting, the Eddington limit also limits the accretion rate. Comparing the spherical accretion luminosity  $L_{\text{acc}} = GMM\dot{M}/R$  to the Eddington limit yields  $\dot{M} \approx 10^{-2} M_{\odot}/\text{yr}$  for  $100 M_{\odot}$ .



**Disks:** a solution to the radiation pressure problem is the formation of a system composed of an accretion disk and polar outflows. The accretion disk allows matter to be fed to the forming star, while the outflows provide an escape for radiation pressure.

## Photoionization

**HII regions:** a UV photon with  $\lambda < 912 \text{ \AA}$  can ionize a hydrogen atom from the ground level ( $n = 1$ ). O-B stars (high enough temperature), emit UV photons, and ionize the ISM around them, forming HII regions. Typical temperatures: 6000 K.

**Strömgren spheres:** consider a sphere of radius  $R_S$  (*Strömgren sphere*) where there is ionization and recombination. Recombination rate  $\propto n_p$  and  $\propto n_e$ , so, recombination rate  $= \alpha n_p n_e$ , where  $\alpha$  is the recombination coefficient ( $n = N/V$ ). In a steady state, number of ionizations = number of recombinations, and so, the total number of ionizations within the Strömgren sphere is  $(4\pi/3)R_S^3 \alpha n_p n_e$ . The star emits  $N_\gamma$  ultraviolet photons that produce the ionizations, so,  $N_\gamma = (4\pi/3)R_S^3 \alpha n_p n_e$ .

**Emission from recombination:** if in the recombination, the free electron is captured and jumps to  $n = 1$ , then a UV photon is emitted. However, this can also happen in stages: first, the free electron is captured to  $n > 1$ , and then it jumps again to  $n = 1$ . This produces radiation in visible light and even in radio. Hot gas from HII regions also emits bremsstrahlung.

**Hypercompact HII regions:** when a massive protostar gains enough mass to go into the main sequence it's still accreting, but starts emitting ionizing radiation. The ionized region is small, because of the infalling gas. Typical scales: size  $< 0.01 \text{ pc}$  ( $=2000 \text{ au}$ ); number density  $\sim 10^6 \text{ e}^-/\text{cm}^3$ . It can be roughly obtained by thinking of the gravitational radius (radius for which sound waves can't escape)  $r_g = GM_\star/c_i^2$ , where  $c_i \approx 10 \text{ km/s}$  is the sound speed of the ionized gas.

**Ultracompact HII regions:** as the collapse from the cloud progresses, the density of the infalling envelope decreases (ram pressure decreases) and the star, having gained more mass, increases the ionizing flux. Typical scales: size  $\sim 0.1 \text{ pc}$  (size of the cloud core), number density  $\sim 10^4 \text{ e}^-/\text{cm}^3$ . These scale can be derived from the Strömgren radius.

**Later stages:** when  $R_S \gg r_g$ , the HII region is not gravitationally bound and it begins to ionize the molecular cloud material in the neighborhood. The disk becomes photoionized and starts to photoevaporate, which may have an impact on the final mass of the star. If the HII region is too big (escape from the cloud core), the high temperatures ( $\sim 8000 \text{ K}$ ) expand the gas further and rapid mass loss from the cloud can occur, even destroying the star-forming region.

## Magnetic fields in diffuse media

**Mass to flux ratio:** consider a spherical cloud of mass  $M$ , radius  $R$ , threaded through a uniform magnetic field of magnitude  $B$  along an axis (let's choose  $z$ ). The gravitational energy is  $\sim GM^2/R$ , and the magnetic energy is  $\sim B^2/(8\pi) \cdot (4/3 \pi R^3)$  in Gaussian units. In equilibrium for the gravitational potential, the virial theorem holds:  $\langle U \rangle + 2\langle K \rangle = 0$ . If the cloud is supported by magnetic energy, instead of kinetic energy, we can expect a similar relation:  $GM^2/R \sim B^2 R^3 \implies GM^2 \sim B^2 R^4$ . But the magnetic flux through the cloud = magnetic flux through the midplane =  $\Phi_B = B(4\pi R^2) \sim BR^2$ , which means  $GM^2 \sim \Phi_B^2$  or  $(M/\Phi_B)_{\text{crit}} \sim 1/\sqrt{G}$ , where we added the subindex "crit" to signal that this is the *critical* value, i.e., the value for avoiding collapse with magnetic energy. The *normalized mass to flux ratio*  $\bar{\mu}$  is the ratio of the mass to magnetic flux with respect to the critical value:  $\bar{\mu} = (M/\Phi_B)/(M/\Phi_B)_{\text{crit}}$ .

**Ambipolar diffusion:** in a plasma, forces do not move ions and neutral particles the same way, because the Lorentz force only applies to ions. This should make ions drift from neutral particles (*ambipolar drift*). However, there is friction (collisions), which eventually moves the ions, although it also dissipates heat. This process is called ambipolar diffusion: it is a momentum redistribution mechanism that allows magnetic forces to act on neutral particles.

**Ambipolar diffusion in a collapsing cloud:** a cloud does not collapse if the magnetic field is too close to its critical value. Ambipolar drift, however, allows the neutral particles to drift from the ions and some material collapses. This increases the gravitational energy until there cloud becomes super-critical ( $\bar{\mu} > 1$ ) and the gravitational collapse to form a star can proceed.

**Limiting cases:** a) neutrals frozen with ions (e.g., ideal MHD): no ambipolar drift, full effect of ambipolar diffusion in the sense of ions being coupled to neutrals, but no diffusion in the sense of the magnetic field being decoupled from the flow. b) Neutrals completely decoupled from ions: full ambipolar drift, no ambipolar diffusion  $\implies$  two independent fluids. c) Neutrals coupled somewhat to ions: some ambipolar drift, ambipolar diffusion with heat dissipation (because of the drift,  $v$  is not frozen into  $B$  and hence diffusion; if it's one fluid, this is called the *strong coupling approximation*). Although  $B$  drifts from  $v$ , reconnecting magnetic field lines requires Ohmic resistivity (or other physical effects).

**Ambipolar diffusion in more detail:**

*Collision rate:*  $\gamma_{pn} = \langle \sigma v \rangle_{in} / (m_i + m_n)$  is the collision rate coefficient. The subindices are: ions (i), plasma (p) and neutrals (n). Typical example for cold interstellar gas where the ions are metals:  $\langle \sigma v \rangle_{in} = 1.9 \cdot 10^{-9} \text{ cm}^3 \text{ s}^{-1}$  [Draine

et al 1983]. For the solar chromosphere, the gas is warm and protons are the dominant ion species → De Pontieu et al 2001.

*Modifications to MHD for a partially ionized plasma:* the continuity equation splits in two (one per species), and it is no longer equal to zero because the species ionize and recombine. The momentum equation also splits in two (one per species) and the collisions between ions and neutrals provide extra momentum that must be taken into account. The induction equation only affects the plasma.

*One fluid approximation:*  $\vec{u} = (\rho_p \vec{u}_p + \rho_n \vec{u}_n) / \rho$  (velocity of the center of mass),  $\rho = \rho_p + \rho_n$ ,  $\vec{u}_D = \vec{u}_p - \vec{u}_n$  is the drift velocity. Valid on large lengthscales and long timescales, where the plasma and neutrals are coupled dynamically and thermally, and close to ionization equilibrium, and if the medium is weakly ionized, so that the inertia of the plasma is negligible. With the approximation, the momentum equation of the plasma implies simply that  $\vec{u}_D = (\vec{J} \times \vec{B}) / (\rho_p \rho_n \gamma_{pn} c) = ([\vec{\nabla} \times \vec{B}] \times \vec{B}) / (\rho_p \rho_n \gamma_{pn})$ , and the magnetic induction equation becomes  $\partial_t \vec{B} = \vec{\nabla} \times (\vec{u} \times \vec{B}) + \vec{\nabla} \times (\vec{u}_D \times \vec{B})$ .

*Ambipolar diffusion timescale:* for a magnetically supported cloud, the Lorentz force balances gravity, and so,  $JB/c \sim GM\rho/R^2 \sim 4\pi G\rho^2 R/3$ . In a molecular cloud,  $m_i \gg m_n$ . We define the ionization fraction as  $x_i = n_i/n_n$ . So, the drift velocity becomes  $u_D \sim \frac{4\pi G\rho^2 R}{3\rho^2 \gamma_{pn}} \sim \frac{4\pi GRm_i}{3\langle\sigma v\rangle_{in}} \sim \frac{4\pi GRm_n}{3x_i\langle\sigma v\rangle_{in}} \implies \tau_{AD} \sim \frac{R}{u_D} \sim \frac{3x_i\langle\sigma v\rangle_{in}}{4\pi Gm_n}$ . For a molecular cloud,  $m_n = 3.9 \cdot 10^{-24}$  g (molecular H, 10% of atoms are He), and  $x_i \sim 10^{-7} \implies \tau_{AD} \sim 10^6$  yr, which is the timescale over which a cloud would lose magnetic support due to ambipolar drift.

*Ambipolar diffusivity:* approximating  $\nabla \rightarrow 1/L$ , the drift velocity becomes  $u_D \sim \frac{B^2/L}{\rho_n} \cdot \frac{1}{\rho_p \gamma_{pn}} \sim \frac{v_A^2}{L} \cdot \tau_{ni}$ , where  $\tau_{ni} = 1/(\rho_p \gamma_{pn})$  is the neutral-ion collision time and  $v_A$  is the Alfvén velocity. The numerator has dimensions of diffusivity, and so, the ambipolar diffusivity is defined as  $\lambda_{AD} = v_A^2 / (\rho_p \gamma_{pn})$ .

*Ambipolar Reynolds number:* in analogy with Ohmic resistivity, we define  $R_{AD} = Lu / \lambda_{AD}$ .

## References

- Zinnecker & Yorke 2007 ARA&A 54 481
- Bodenheimer 2011 *Principles of Star Formation*, Springer
- Fleishman & Toptygin 2013 *Cosmic Electrodynamics*, Springer
- Stahler & Palla 2004 *The formation of Stars*, Springer
- Kuiper, Klahr et al 2014 In *The Labyrinth of Star Formation*, ed. Stamatellos, Goodwin & Ward-Thompson
- E. G. Zweibel 2015 In *Magnetic Fields in Diffuse Media*, Springer 2015

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