para Introducción a la Física Espacial

Radiation and matter

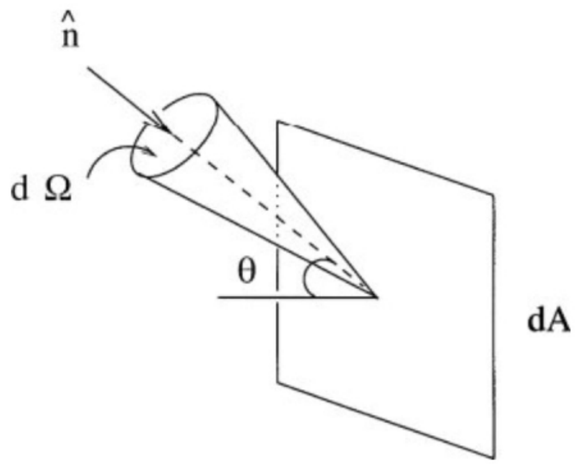
Contents

Radiative transfer

Quantity	Definition	Units
Luminosity/ power	(SB Law: $L = \sigma A T^4$)	W
Flux	$F = L/A$	$\mathrm{W\,m^{-2}}$
Intensity	$dF = I \cos\theta d\Omega$	$\mathrm{W\,m^{-2}\,sr^{-1}}$

Basic quantities of radiative transfer

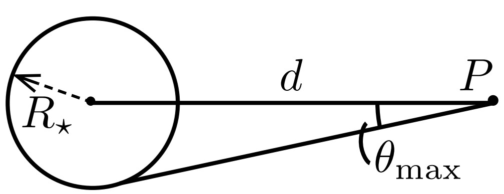
Blackbody radiation:
Planck's law: the energy density U_ν for the frequency ν (formally, the frequency range, $\nu, \nu + d\nu$) is



$$U_\nu d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp\left[h\nu/(k_B T)\right] - 1} := \frac{4\pi}{c}B_\nu(T)$$

Specific intensity. The energy of a given frequency coming from a solid angle $d\Omega$ and hitting over an area dA (projected perpendicularly to it) in a time dt is $dE_\nu d\nu = I_\nu(\vec{\mathbf{r}}, t, \hat{\mathbf{n}})\cos\theta\, dA\, dt\, d\Omega\, d\nu$, where I_ν is the specific intensity

.**Radiation flux for a frequency ν:**
$F_\nu = \int I_\nu \cos\theta d\Omega$ (power that crosses an area from all directions). The total flux for all frequencies is $F = \int F_\nu d\nu$.



Uniformly radiating sphere: consider a sphere of radius $R_\star$ that radiates with uniform intensity I_0. z-axis along d. The flux that an observer located at P receives is $F = \int I \cos\theta d\Omega$

$$= I_0 \int_0^{2\pi} d\phi \int_0^{\theta\max} \cos\theta \sin\theta d\theta = I_0 \cdot \frac{1}{2}\sin^2\theta_{\max} \cdot 2\pi,$$

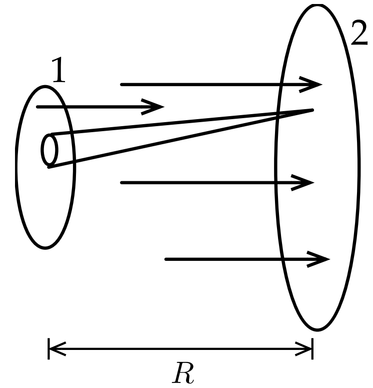
but since $\sin\theta_{\max} = R_\star/d, \implies F = I_0\pi(R_\star/d)^2$. Observe that the term $\pi(R_\star/d)^2$ is the solid angle subtended by the sphere at P (both making the star bigger or closer have the same effect).

Energy density: energy dE_ν that passes through a cylinder of base dA and height cdt:
$$\frac{dE_\nu}{\cos\theta dA\, cdt} = \frac{I_\nu}{c}d\Omega \implies U_\nu = \int \frac{I_\nu}{c}d\Omega$$; if isotropic, $U_\nu = \dfrac{4\pi}{c}I_\nu$

Radiation pressure. Radiation has momentum dE_ν/c.
$$\text{pressure} = \frac{\text{force}_\perp}{dA} = \frac{\text{momentum}}{dtdA} = \frac{dE_\nu\cos\theta}{c}\frac{1}{dAdt} = \frac{I_\nu}{c}\cos^2\theta d\Omega$$
. The pressure counting all directions is $P_\nu = \dfrac{1}{c}\int I_\nu \cos^2\theta d\Omega$; if isotropic, $P_\nu = \dfrac{4\pi}{3}\dfrac{I_\nu}{c}$.
Comparing with the isotropic energy density, $P_\nu = \dfrac{1}{3}U_\nu$.

Radiative transfer in empty space: the radiation going through 2 due to 1 is $I_{\nu 2} dA_2 dt d\Omega_2 d\nu$, and, by symmetry, the radiation that 1 receives due to

2 is $I_{\nu 1} dA_1 dt d\Omega_1 d\nu$. But the energy is conserved, and $d\Omega = dA/R^2$, so, $I_{\nu 1} = I_{\nu 2}$, which implies that, along the ray path, $\dfrac{dI_\nu}{ds} = 0$ in empty space.



Conclusion: the specific intensity does not depend on distance, so it's a measure of surface brightness (total $I \propto r^{-2}$ but the angular size also is $\propto r^{-2}$, so the effects cancel out).

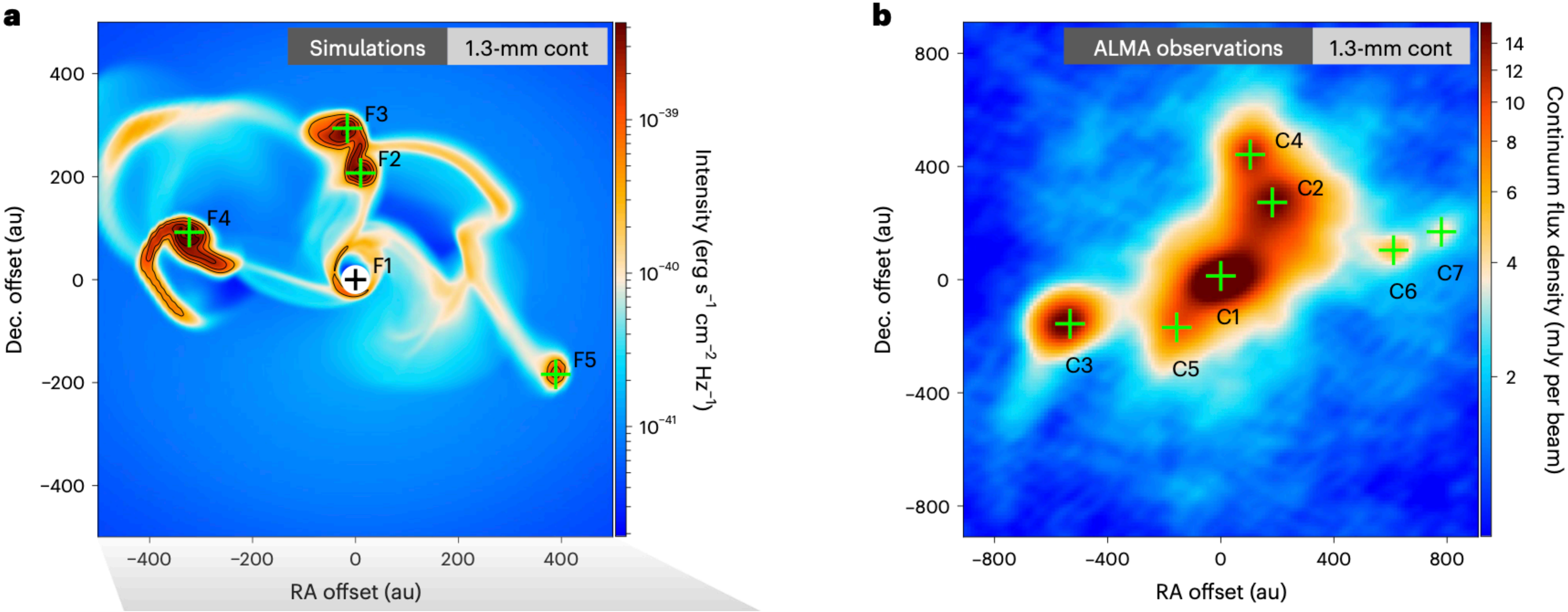
Radiative transfer equation: in the presence of matter, $\dfrac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$. If matter emits, it sums j_ν (emission coefficient, units of intensity/length). If matter absorbs, it will diminish the intensity, so one subtracts an amount proportional to I_ν; α_ν is the absorption coefficient.

Optical depth: if we consider absorption only, $dI_\nu/ds = -\alpha_\nu I_\nu \implies dI_\nu/I_\nu = \alpha_\nu ds$. Integrating in a path, we get $\ln\left[\dfrac{I_\nu(s)}{I_\nu(s_0)}\right] = -\displaystyle\int_{s_0}^{s} \alpha_\nu(s')ds'$. We define the integral in the rhs as the optical depth τ_ν. Then, the solution for only absorption is $I_\nu(s) = I_\nu(s_0)e^{-\tau_\nu}$. If the optical depth is $\ll 1$, the intensity at the beginning is almost the same as at the end $\implies$ the medium is optically thin or transparent. If the optical depth is $\gg 1$, the final intensity is about zero $\implies$ the medium is optically thick or opaque.

Source function: we define $S_\nu := j_\nu/\alpha_\nu$. We can have the radiative transfer equation in terms of S_ν, τ_ν: $\dfrac{1}{\alpha_\nu}\dfrac{dI_\nu}{ds} = \dfrac{j_\nu}{\alpha_\nu} - I_\nu \implies \dfrac{dI_\nu}{d\tau_\nu} = S_\nu + I_\nu$. This differential equation can be solved by using an integrating factor e^{τ_ν}: $\dfrac{d}{dt}(I_\nu e^{\tau_\nu}) = S_\nu e^{\tau}_\nu$; we integrate from $s_0 < s' < s$ in the ray path, which corresponds to $0 < \tau'_\nu < \tau_\nu$, yielding $I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \displaystyle\int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu$. $I_\nu(0)$ is the boundary condition and can be fixed by including incoming radiation. $I_\nu(\tau_\nu)$ is the intensity emerging from the medium to the observer; the first term is absorption along the ray, and the second term is the sum of the contributions of all sources along the ray.

Limiting cases: for constant coefficients and no incoming radiation, $I_\nu(\tau_\nu) = S_\nu(1 - e^{-\tau_\nu})$. If the object is optically thin, $\tau_\nu \ll 1$, $e^{-\tau_\nu} \approx 1 - \tau_\nu$, with $\tau_\nu = \alpha_\nu L$, being L the length of the material $\implies I_\nu = j_\nu L$. If the object is optically thick, $\tau_\nu \gg 1 \implies I_\nu = S_\nu$.

Kirchhoff's law: when matter is in thermodynamic equilibrium, all that is emitted must be absorbed. That means that $j_\nu = \alpha_\nu I_\nu$, but since $I_\nu = B_\nu$ for equilibrium, $S_\nu = j_\nu/\alpha_\nu = \alpha_\nu I_\nu/\alpha_\nu = B_\nu(T)$.



Looking through a dense molecular cloud

The protostars contained in this accretion disk emit blackbody radiation. This radiation is greatly absorbed by the gas and dust present in the molecular cloud, but not in all wavelengths by the same amount. In visible light, the medium is opaque, but not in these wavelengths detected by ALMA.

Li, Beuther, Oliva et al 2025 Nat Astron

Opacities for stellar material

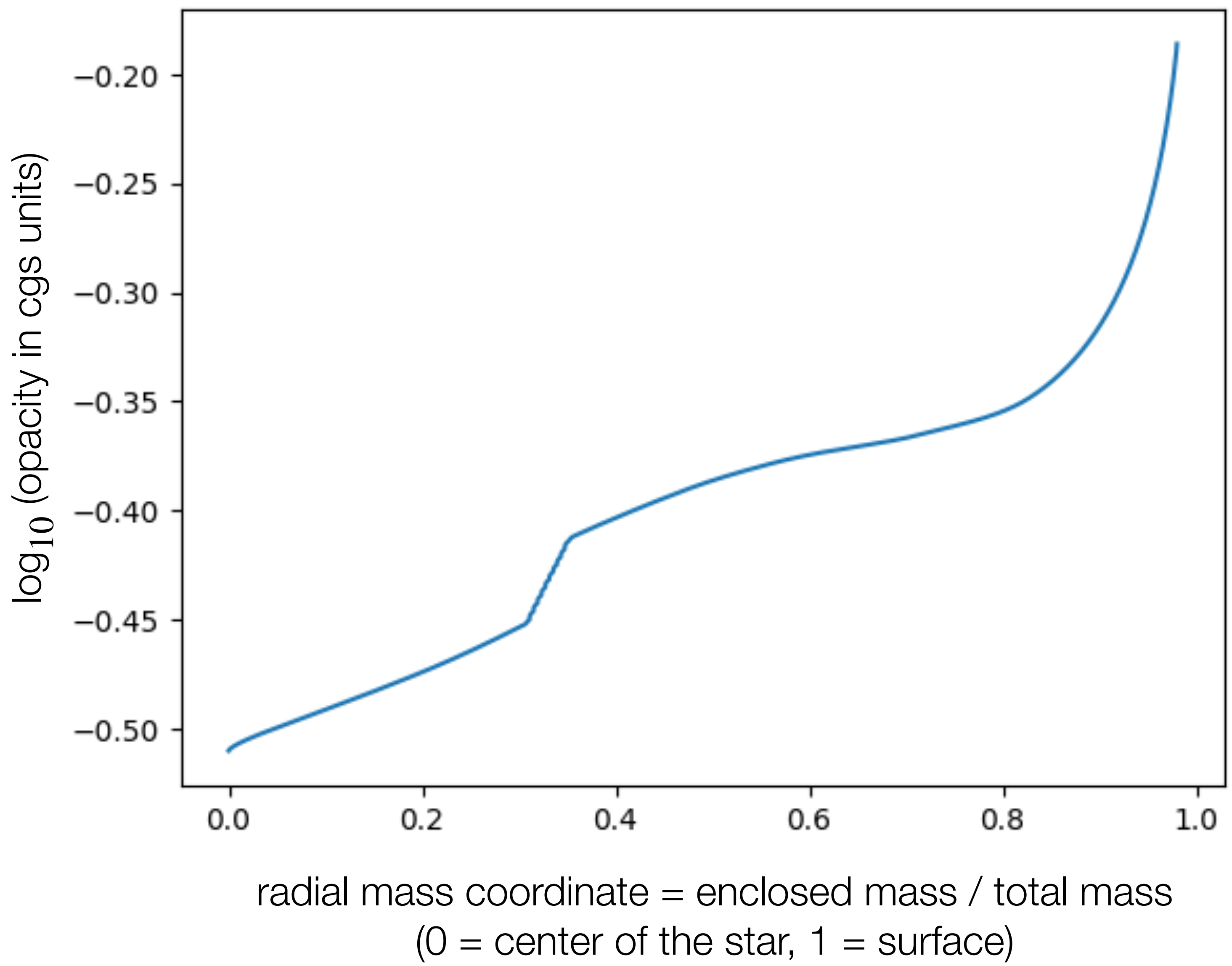
Saha equation (conceptual): it describes ionization number densities from all possible initial states (ionization from the ground state, form the first excited state, from the second excited state, etc.) as a function of temperature. E.g., one can calculate that for temperatures of 10^5 K, hydrogen is almost fully ionized.

Calculations of opacity: based on the number of atoms and electrons in various energy levels (Boltzmann distribution) and various stages of ionization (Saha equation), one can in principle compute the opacity by computing absorption cross-sections for different processes. Since atomic energy levels can be bound (discrete) or free (continuum), absorption of radiation can be due to three different kinds of transitions: bound-bound, bound-free and free-free. The sum of all cross sections plus the effect of stimulated emission gives the absorption coefficient and the opacity. Free-free absorption is the "inverse" of Bremsstrahlung (free-free emission), and it's caused by a free electron gaining energy during a collision with an ion by absorbing a photon.

Kramer's law: with approximations, and for high temperatures ($T \gtrsim 10^5\,\mathrm{K}$), opacity goes like $\chi \propto \rho T^{-7/2}$ (bound-free, free-free). Higher temperature $\Longrightarrow$ less remaining neutrals $\Longrightarrow$ less chances to photoionize $\Longrightarrow$ low opacity. For low T (below $\sim 10^4\,\mathrm{K}$), most atoms are in their lowest energy levels. Photons don't have enough energy to knock electrons off the atoms, so not much radiation is absorbed and opacity drops as well (=transparency). $\therefore$ Opacity for solar metallicity peaks at $T \sim 10^5\,\mathrm{K}$.

Thomson scattering and opacity: For very high temperatures (fully ionized gas), Thomson scattering starts happening (free electrons scatter photons). The extinction coefficient is $n_e\sigma_T$ (where σ_T is the Thomson cross section, i.e., how the photon sees the electron) and the opacity is then $\chi_T = n_e\sigma_T/\rho$. The classical cross section calculation of σ_T (electron in a oscillating electric field) depends on frequency and reduces to σ_T for very high frequencies. For low frequencies where the electron is tightly bound to an atom (behaving like a spring with freq. ω_0), one gets Rayleigh scattering, $\sigma_R = \sigma_T(\omega/\omega_0)^4$

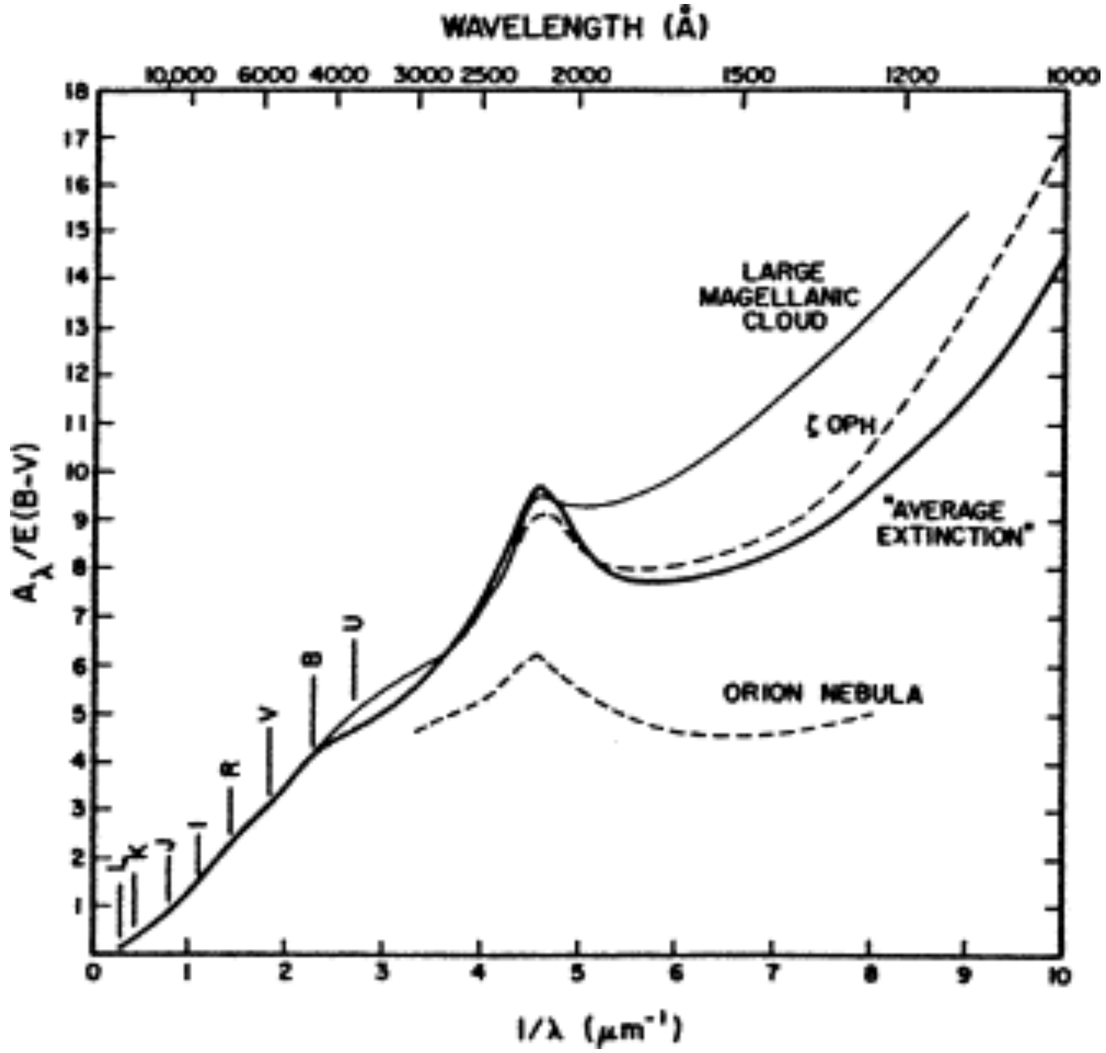
Solar surface opacity: the solar surface is opaque, but the black body radiation (at $T \sim 6000\,\mathrm{K}$) doesn't have enough energy to ionize H. However, H^- ions are also formed because the electron doesn't fully screen the positive electrostatic force of the nucleus. Breaking this bond brings the required opacity.



Opacity inside of a massive star

In this plot produced with the Geneva stellar evolution code, one can see that the interior of the star (where temperature is high) is more transparent than the surface (see Kramer's law). Opacity changes as the kind of microphysical process that produces absorption changes according to the temperature, density, etc.

Dust opacities and polarization

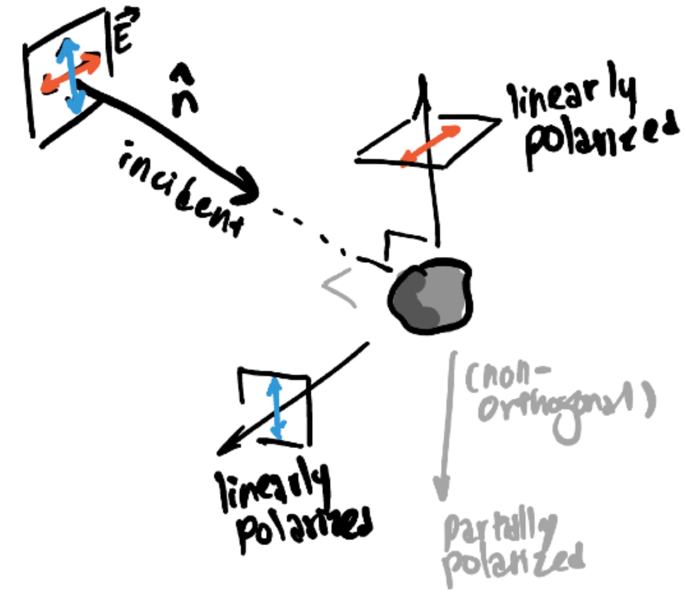


Dust efficiency of extinction: dust grains are big compared to gas molecules, so they contribute more to the extinction of radiation. We can write for the dust $\rho_d \kappa_\nu = n_d \sigma_d Q_\nu$, where σ_d is the cross-section for a typical grain of radius a_d, $= \pi a_d^2$. Q_ν is the extinction efficiency factor and it can be obtained empirically as $\frac{Q_\lambda}{Q_{\lambda_0}} = \frac{A_\lambda / E_{B-V}}{A_{\lambda_0}/E_{B-V}}$, where the subindex 0 means a reference value. Empirically, we have $Q_\lambda = 0.14 A_\lambda / E_{B-V}$ and the ratio A_λ / E_{B-V} is the normalized total extinction. In star-forming regions, the gas is transparent in the far IR and mm wavelengths, so hot dust clouds dominate emission (one observes dust but not gas at those wavelengths).

Dust sizes and composition: observations of dust extinction are consistent with dust particles composed of refractory cores surrounded by icy mantles. The cores are rich in silicates and the mantle is a mixture of water ice and other molecules. In most models, a radius of $a_d \sim 0.1\,\mu$m, but there must be a dust size distribution (for example, the Mattis-Rumpl-Nordsiek distribution has upper and lower cutoffs at 0.25 μm and 0.005 μm).

Dust abundances: consider a HI cloud with hydrogen number density n_H. We define $\Sigma_d = n_d \sigma_d / n_H$ to be the total geometric cross section of grains per hydrogen atom. Given that $1/(\rho\kappa_\nu)$ is the photon mean free path and (rad.transf.eqn.)$\Longrightarrow \rho\kappa_\nu \Delta\ell = \Delta\tau_\nu$ the optical depth, we write $\Delta\tau_\nu = (n_d L)\sigma_d Q_\lambda$ for a column of material of length L. Observationally, one gets $\Sigma_d = 1.0 \cdot 10^{-21}$ cm^2, and with this, $\kappa_\lambda = 420\,\mathrm{cm}^2\,\mathrm{g}^{-1} Q_\lambda$. The dust to gas ratio is computed as $f_d = \frac{4\pi a_d^3 \rho_d}{3\mu m_H}\left(\frac{n_d}{n_H}\right)$ and yields 0.02, but the widely used value is 0.01.

Dust polarization: dust grains are able to polarize radiation by scattering. If there is a stellar source nearby, radiation travels until it hits a dust grain and is scattered. In the directions perpendicular to the incident ray, the radiation is linearly polarized. If there is a magnetic field **B** present in the environment, dust grains (which are irregularly shaped, not really spheres, and have electric charge) will experience a magnetic torque that eventually orients them in the direction of the magnetic field. When radiation from a background source goes through the oriented dust grains, the transmitted radiation will be polarized in the direction of the alignment (= the direction of the magnetic field).



Stellar feedback in Orion

When massive stars photoionize surrounding material, it becomes transparent to radiation. Material from the cloud that is not yet ionized is opaque.

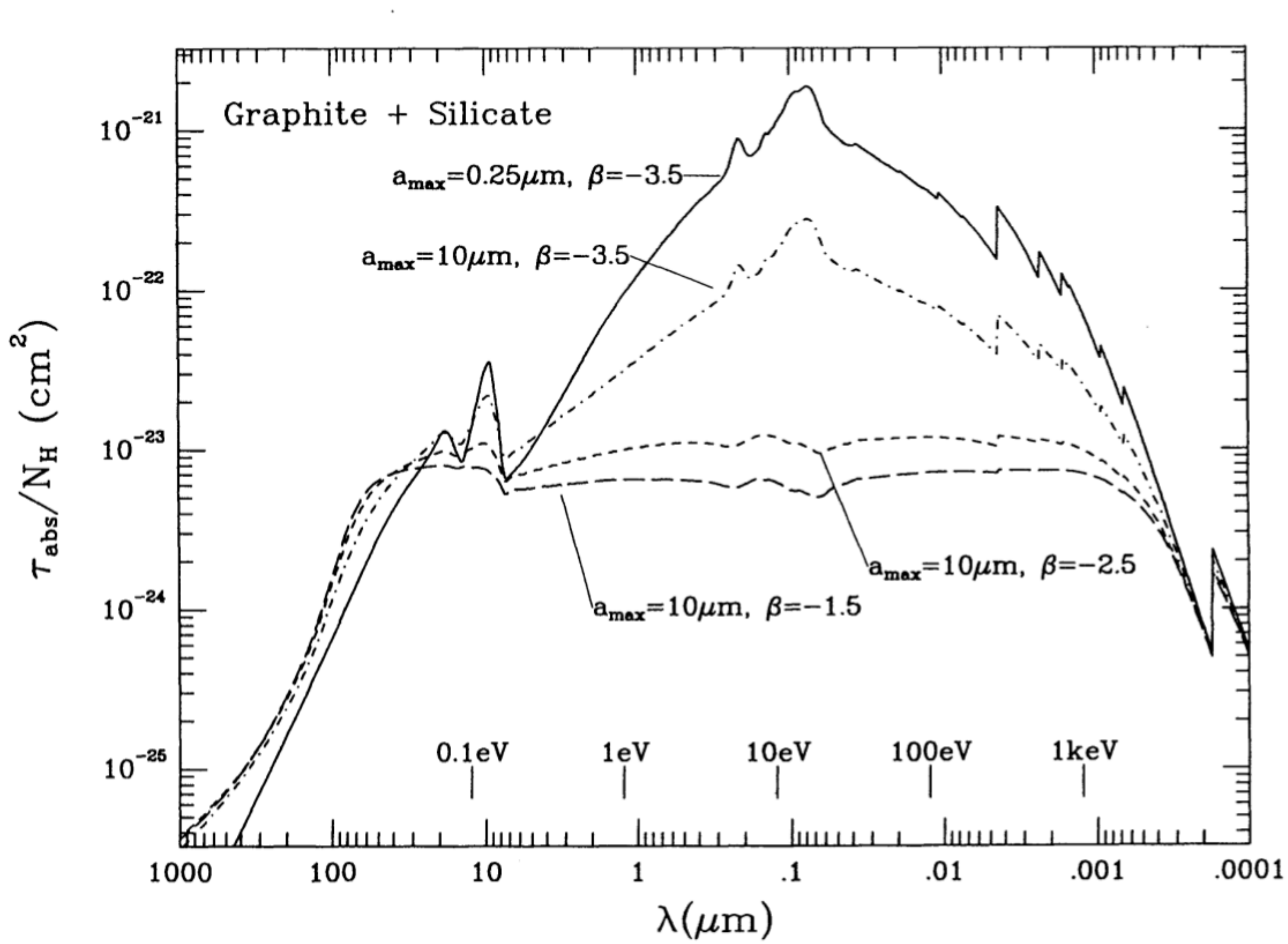
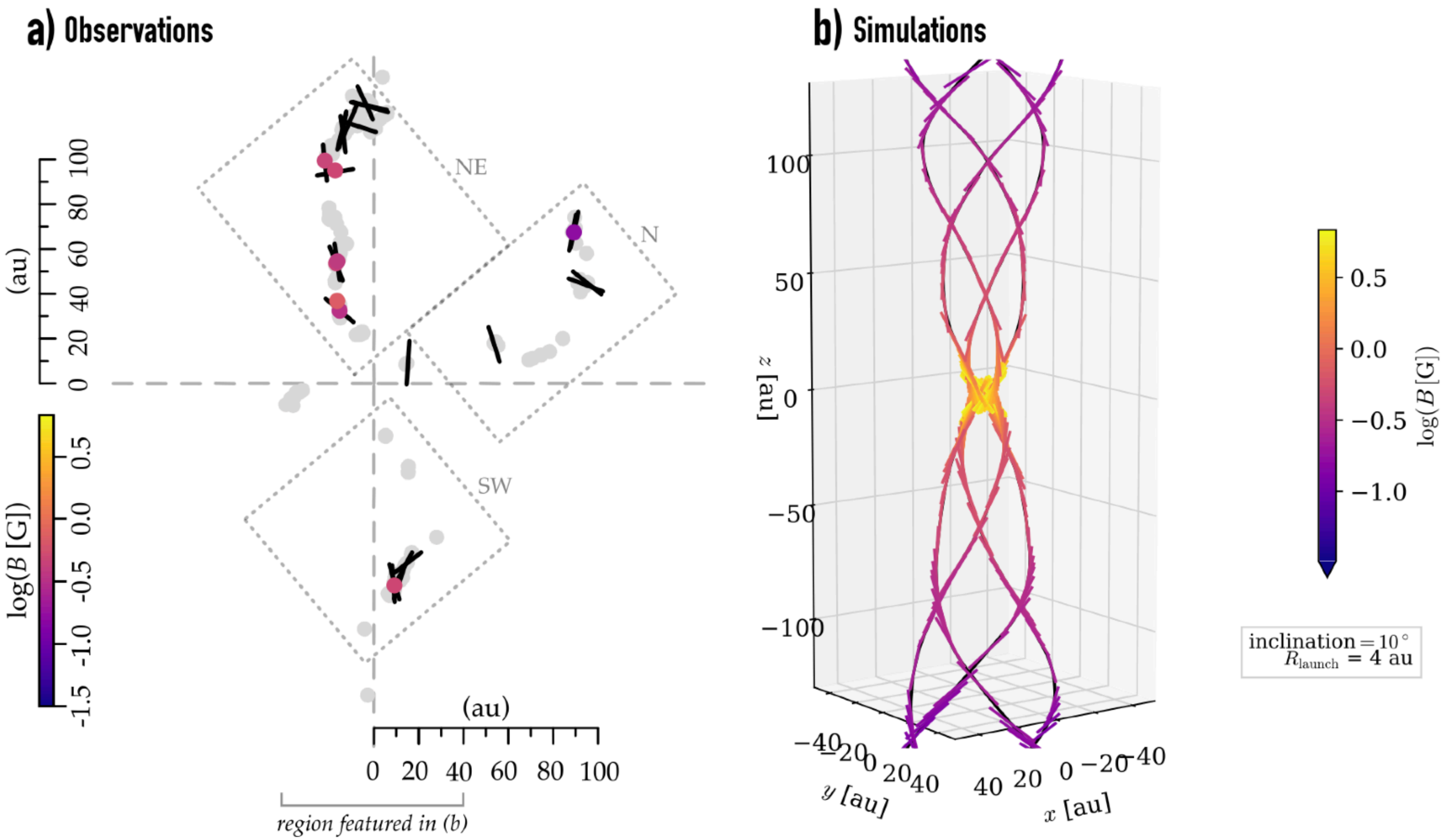


FIG. 6*a*

Dust opacity calculation

In this example paper, dust opacities are computed using different models for the dust grains. The figure shows the absorption cross-section per hydrogen nucleus for dust grains. Note how at large wavelengths, the dust opacity decreases.

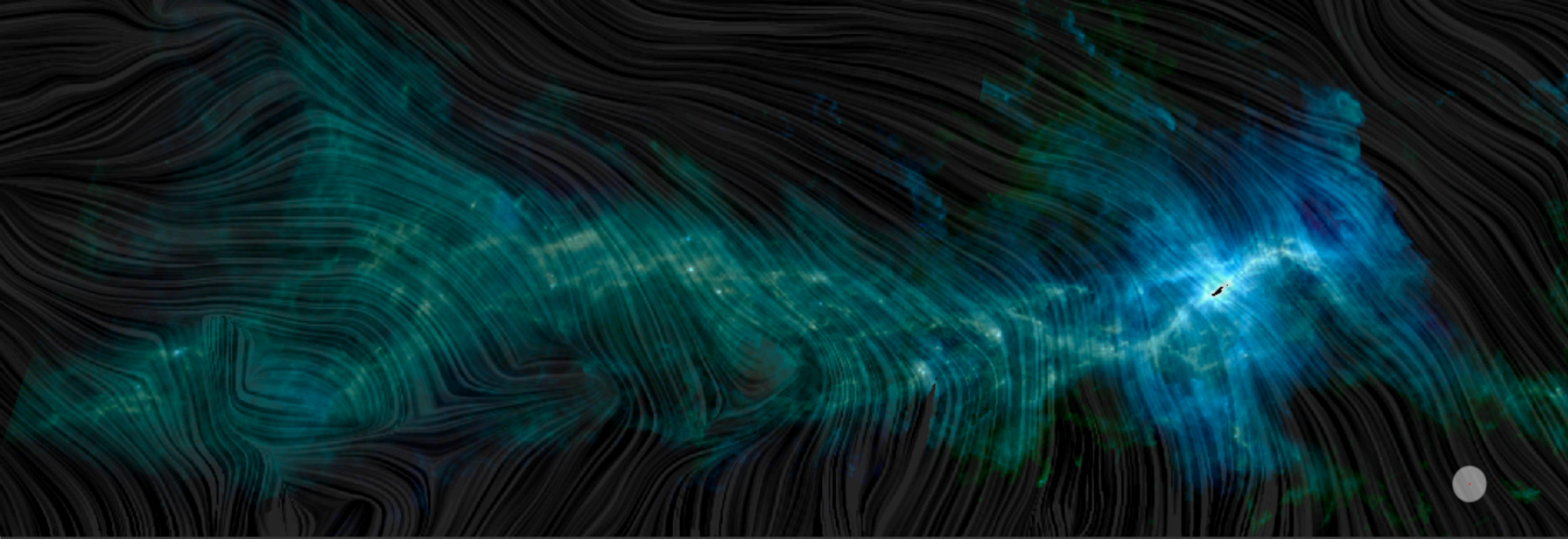
Laor & Draine 1992 ApJ



Probing the magnetic field of a protostellar jet with dust

Thanks to dust polarization, one can study the orientation and strength of the magnetic field of a protostellar jet deep inside of a cloud. Linear polarization allows the study of the orientation of the field, and circular polarization allows for the measurement of the magnetic field strength along the line of sight.

Moscadelli, Oliva et al 2023 A&A



Galactic magnetic fields

In this case, dust polarization was used to measure galactic magnetic fields in Orion A. The lines are the inferred magnetic field perpendicular to the observer's plane.

Soler 2019 A&A